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**MATHEMATICS**

**METHODS**

**UNITS 3 & 4**

**Semester Two**

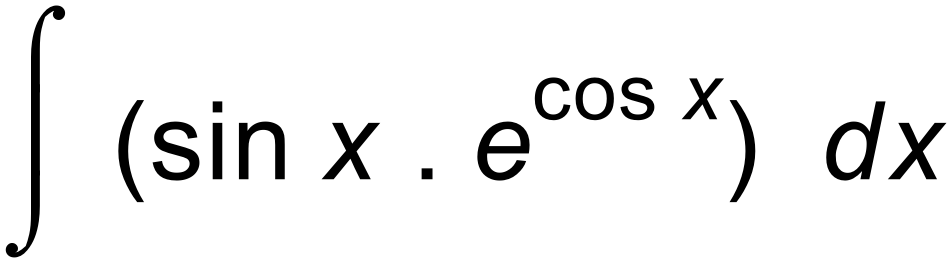
**2017**

**SOLUTIONS**

***Calculator−free Solutions***

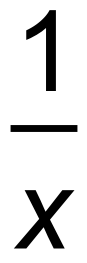
1. (a) 

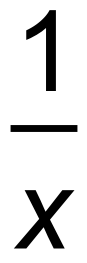
= – sin x e cos x ✓✓

(b) 

= 

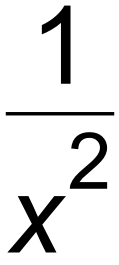
= – e cosx + c ✓✓ [4]

2. (a) f ´(x) = 2e2x –  ✓

For max/min, 2e2x –  = 0 ✓

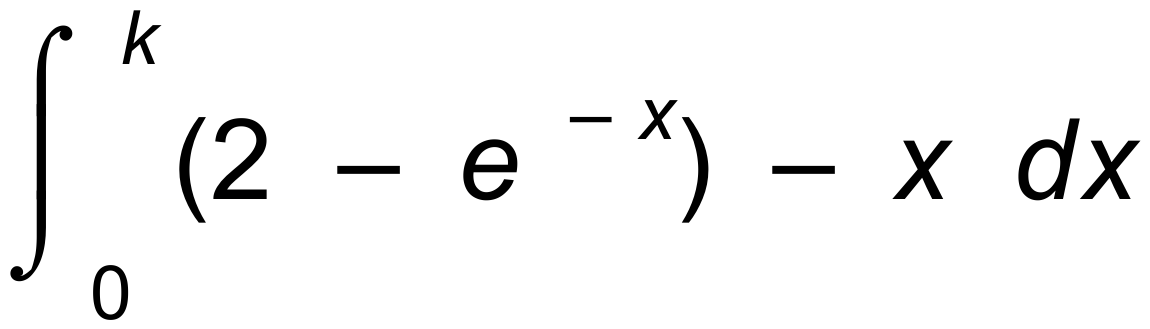
 2xe2x – 1 = 0 ✓

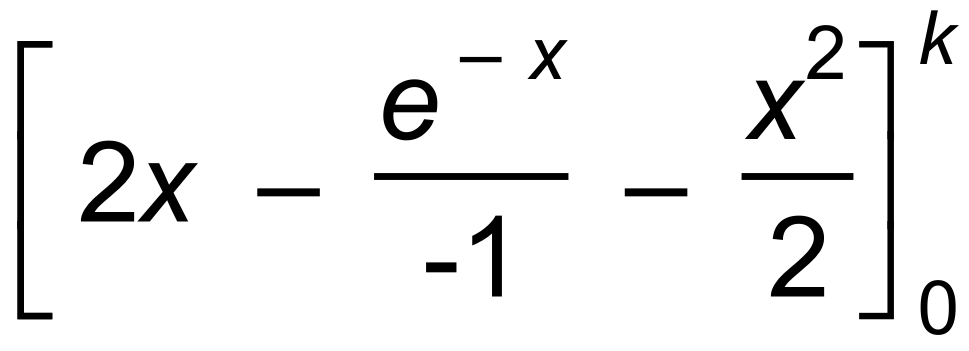
x = 0.5 e –2x

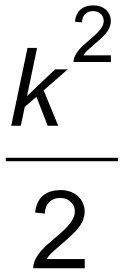
(b) f ′′(x) = 4e2x + 

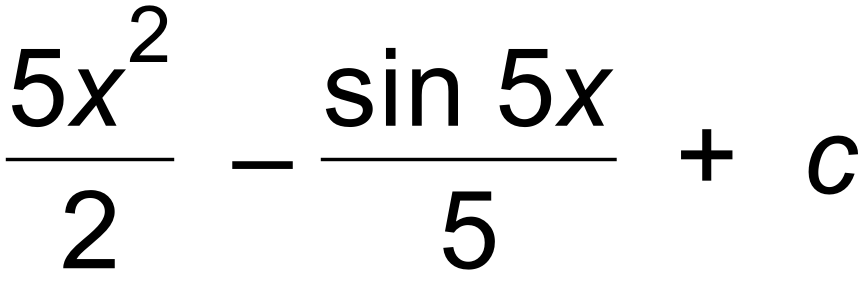
Since expression > 0 for all x values, ✓

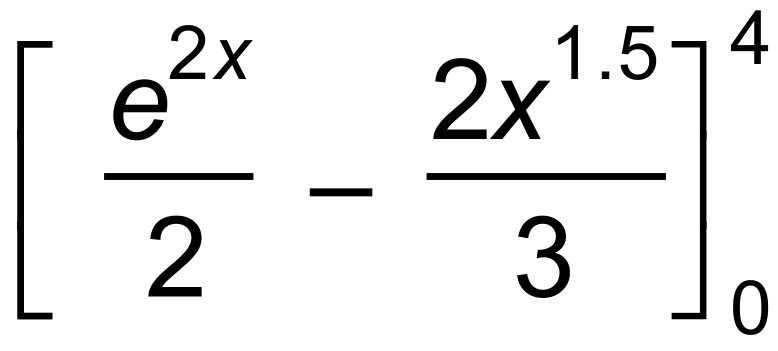
then stationary point is a minimum. ✓ [5]

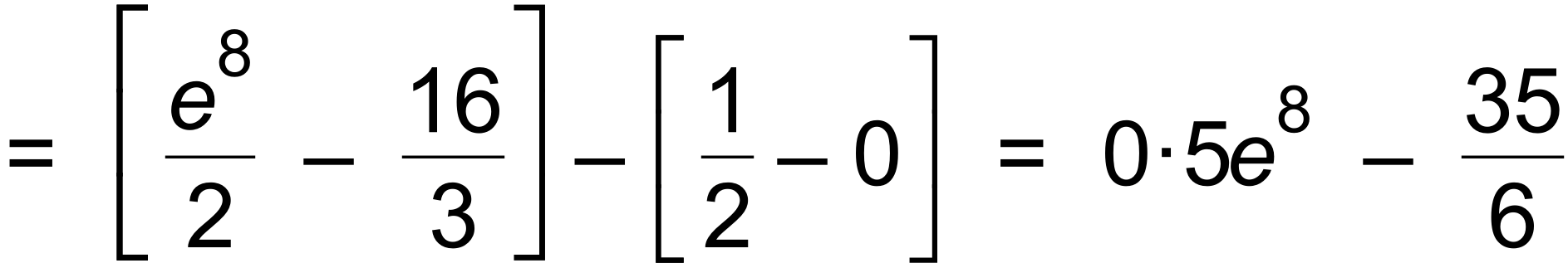
3. (a) A =  ✓✓

(b) A =  ✓

= (2k + e –k –  ) – 1 ✓✓ [5]

4. (a)  ✓✓

(b)  ✓

 ✓✓

(c) 2 sin 2x ✓✓ [7]

5. (a) x = sin 2t + e –2t +c ✓

x(0) = 0+1 + c = 1  c = 0 ✓

x = sin 2t + e –2t ✓

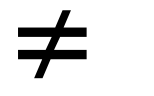
(b) a = –4sin 2t + 4e –2t ✓✓

(c) Assume a = – k2x

Then –4 sin 2t + 4 e –2t = – k2(sin 2t + e –2t)

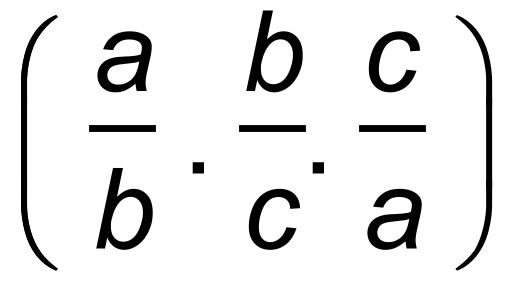
This leads to the result that k2 = 4 and k2 = –4.

Hence, relationship is false.

Or a = –4(sin 2t – 4e –2t) −4(sin 2t + 4e –2t) = −22x

✓✓ [7]

6. (a) log (  ) + log (  ) + log (  )

= log  ✓

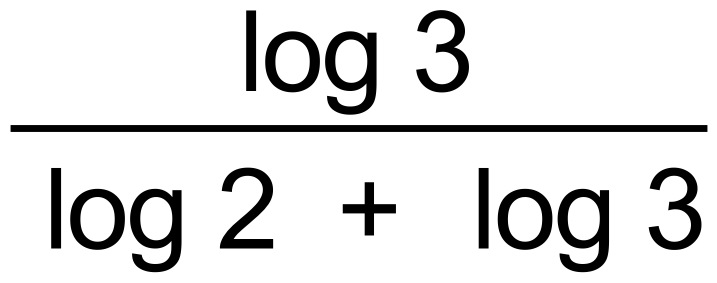
= log 1 = 0 ✓

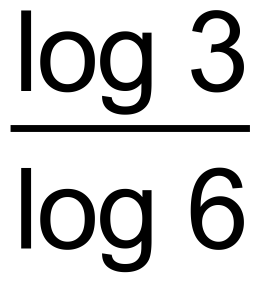
(b) y = 1 – x

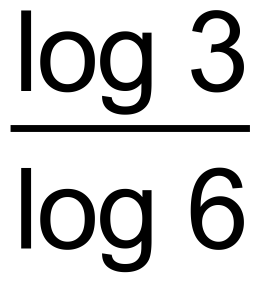
2x = 31 – x

x log 2 = (1 – x)log 3 ✓

x log 2 + x log 3 = log 3 ✓

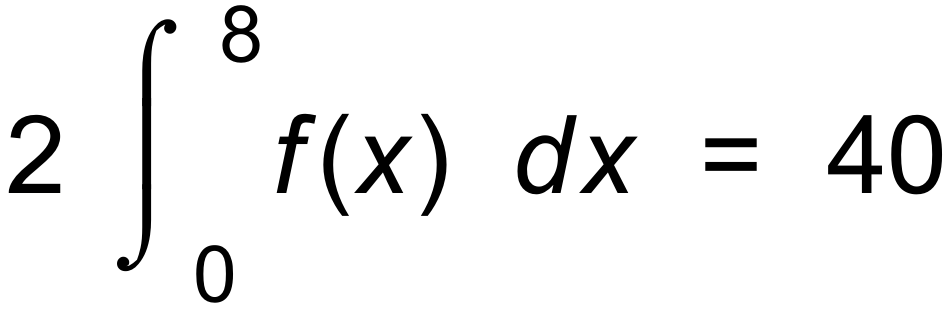
x =  ✓

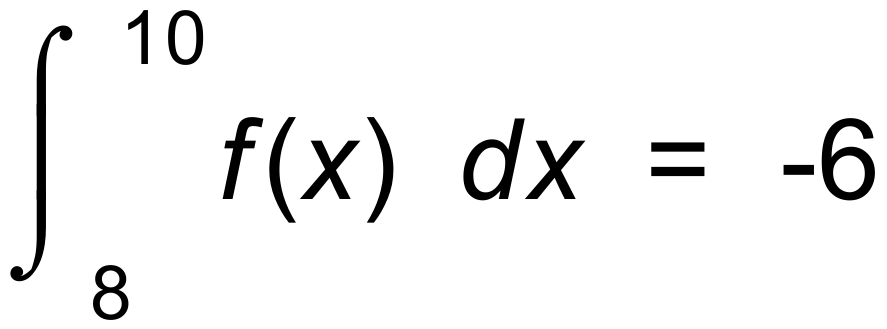
x =  as required ✓

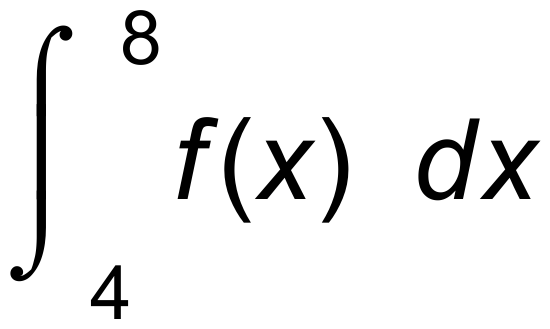
y = 1 –  ✓ [7]

7. (a) (i) 14 ✓

(ii) 14 + 6 – 6 = 14 ✓✓

(iii)  ✓✓

(b)  ✓

 = 6 ✓

 a = 4, b = 8, c = 10 ✓ [8]

8. (a) A = Length x Width = 3xcos 2x ✓

(b) A = 3x cos 2x

A′ = 3x (–sin 2x)(2) + 3cos 2x = 0 for max / min ✓

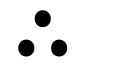
6xsin 2x = 3cos 2x ✓

2x tan 2x = 1 as required. ✓

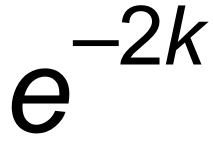
(c) A′′ = –12 sin 2x – 12xcos 2x ✓

= –12(sin 2x + xcos 2x) ✓

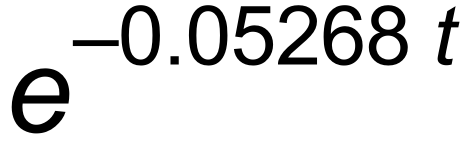
< 0 for maximum ✓

 sin 2x + x cos 2x > 0 as required [7]

***Calculator−assumed Solutions***

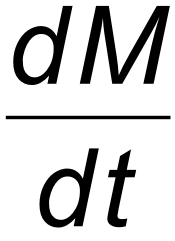
9. (a) Solve 0.9 = 

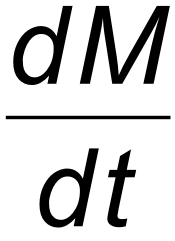
∴ *k* = 0.05268 = 0.0527 ( 3 s.f.) ✓✓

(b) Solve 0.5 = 

*t* = 13.153 ✓✓

Half life is 13.153 years.

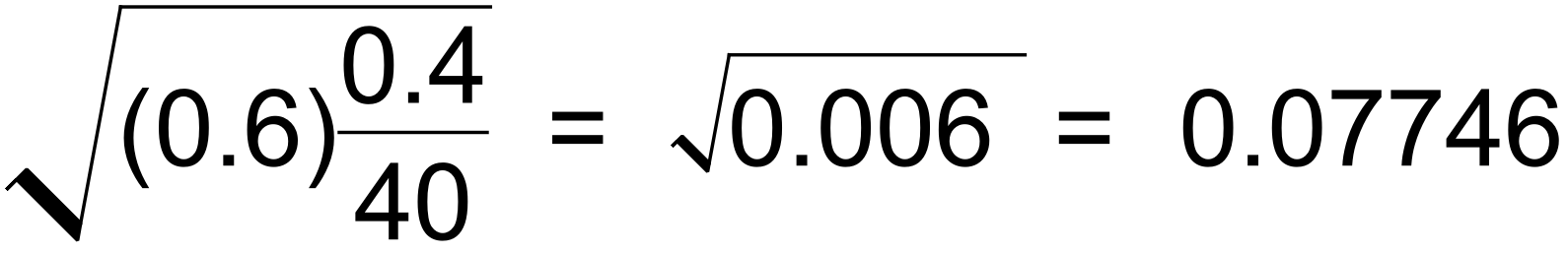
(c)  = *M*0 e –*k**t* . (–*k*) ✓

At *t* = 2,  = 20 . e – 0.0527 (2) . (–0.0527)

= – 0.9486 units of mass per year. ✓✓ [7]

10. (a) Only 2 results for each trial–single or married. ✓

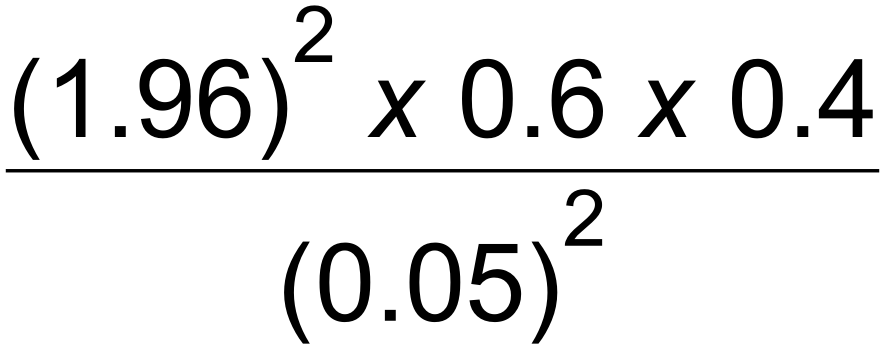
(b) 0.6 ✓

(c)  ✓✓

(d) We can be 95% confident that the true proportion is *p*

where 0.6 –(1.96)( 0.07746) < *p* < 0.6 + (1.96)( 0.07746)

ie 0.4482 < *p* < 0.7518 ✓✓

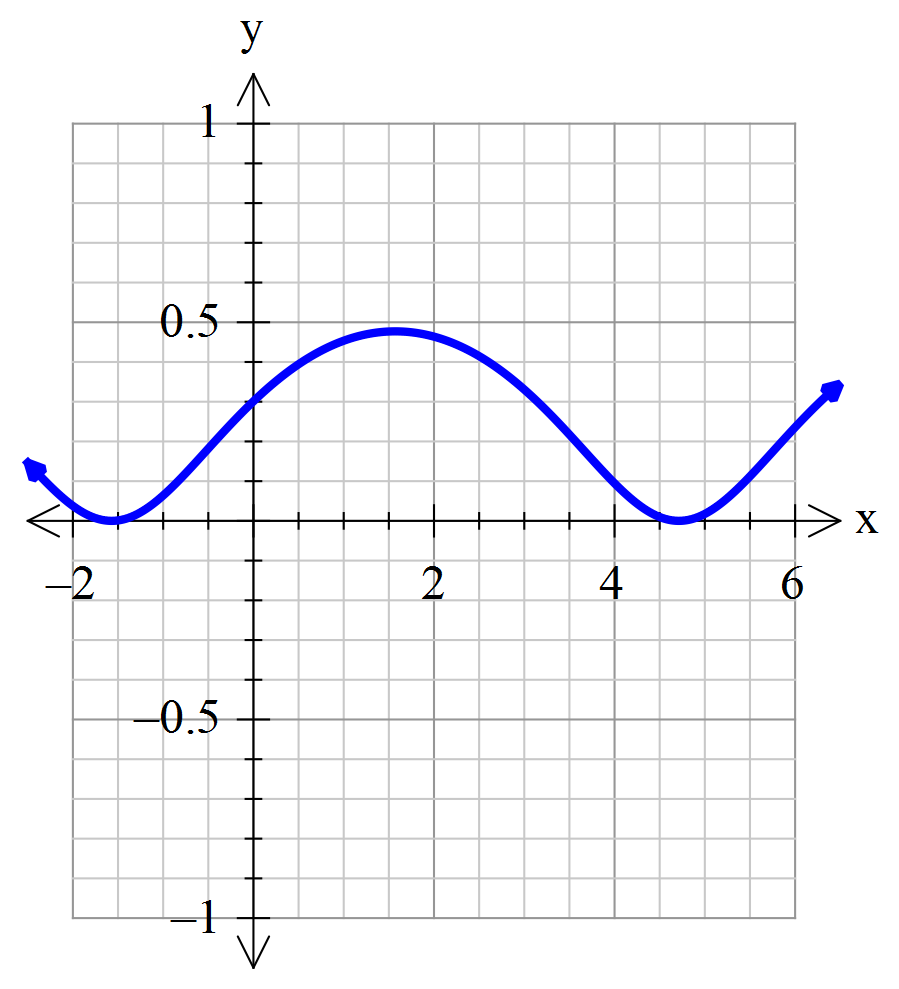
(e) *n* =  ✓

*n* = 368.8 ✓

Sample size needs to be 369. ✓ [9]

11. (a) From calculator, 0.142 ✓

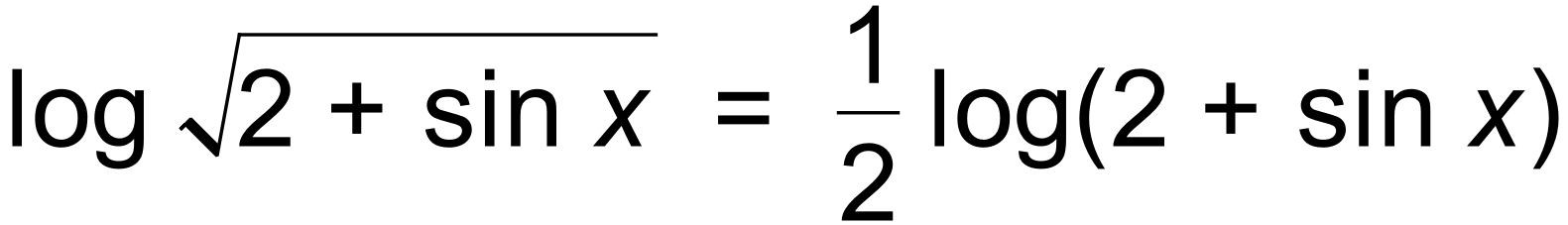
(b) sin *x* has a minimum of –1. So, 2 + sin *x* has a minimum of 1

So log(2 + sin *x*) has a minimum of 0. ✓✓

(c)

✓✓✓

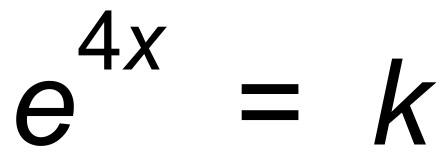
(d) By inspecting the graph, all of this curve is above the *x* axis. ✓

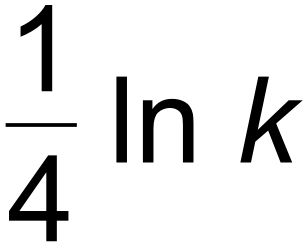
 since  ✓

Area = 0.5(0.142) = 0.071 ✓ [9]

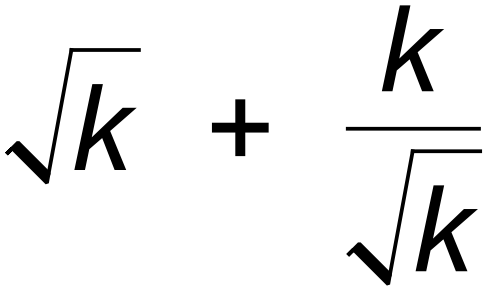
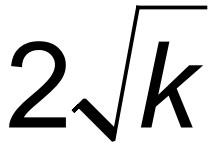
12. (a) Minimum when *f* ′(*x*) = 0

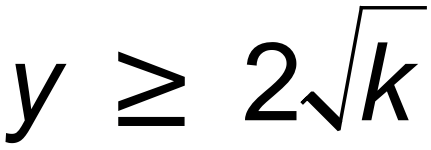
∴ 2  ✓

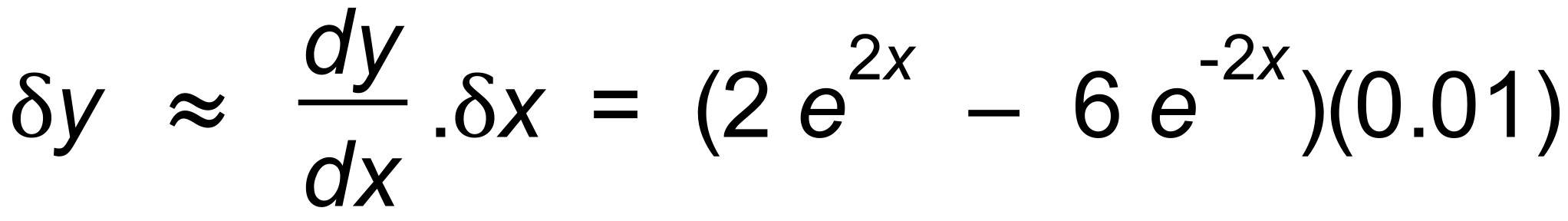
∴ 

∴ *x* =  ✓

∴ Minimum value is 

=  =  ✓

∴ Range is  ✓

(b)  ✓✓

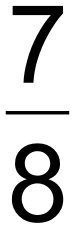
= 1.09 ( 2 decimal places) ✓

(c) *f*(2) = 54.653

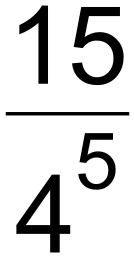
*f*(2.01) = 55.755 ✓

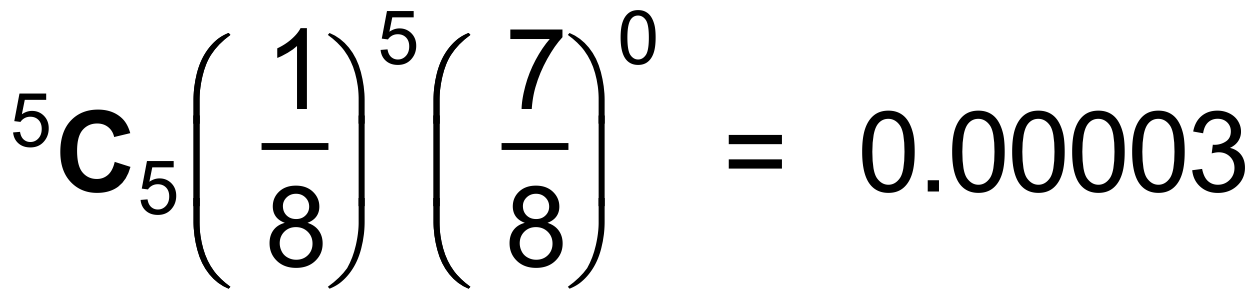
Change is 1.10 ( 2 decimal places) ✓ [9]

13. (a) Not equally likely outcomes, so biased. ✓

(b)  ✓

(c) E(X) = 1.875 Var(X) = 1.05332 = 1.109 ✓✓

(d) P(Y = 4) = 5C4 (0.25)4(0.75) =  = 0.0146 ✓✓

(e) P(five 4s) =  ✓ ✓

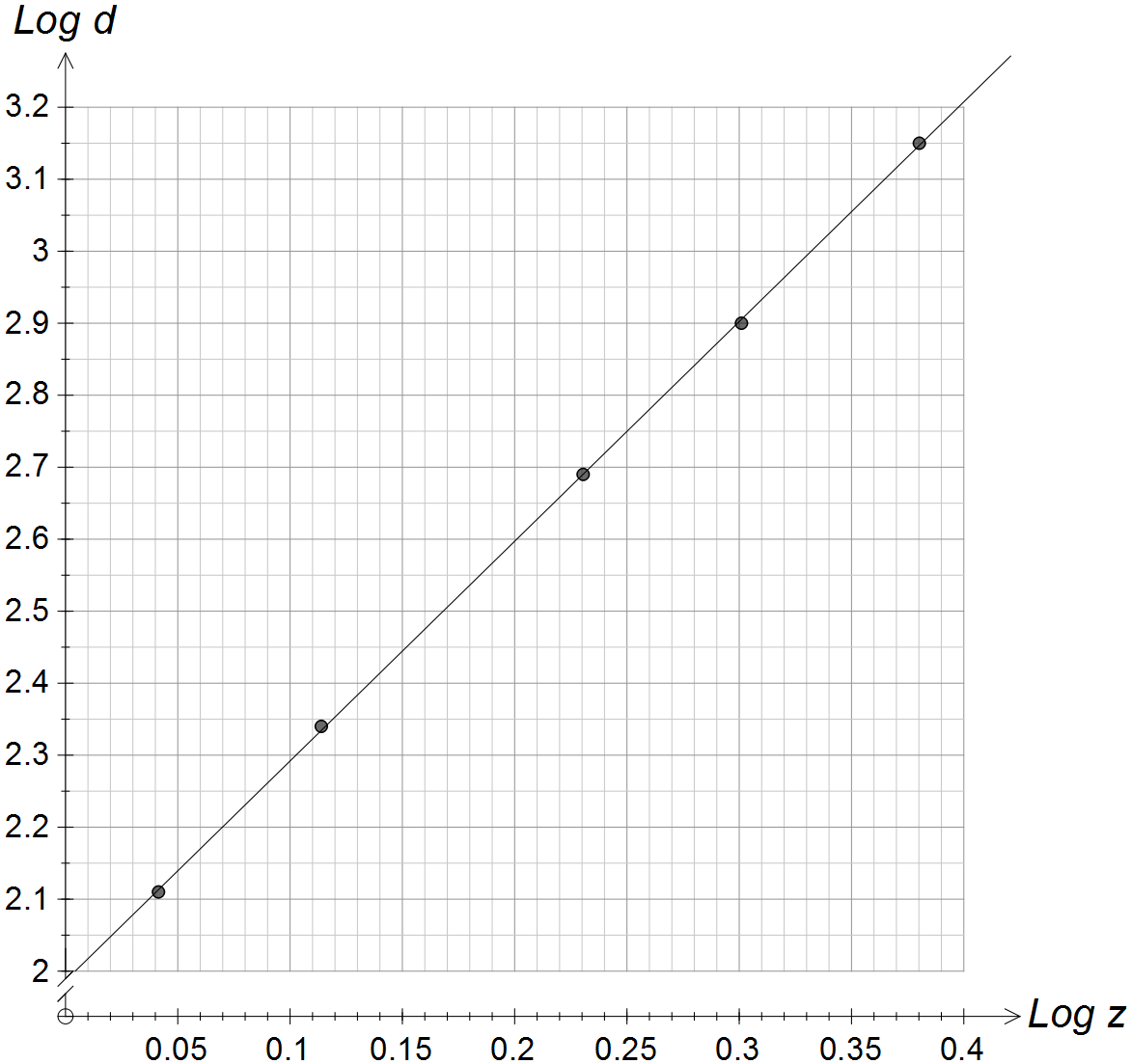
[8]

14. (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *z* | 1.1 | 1.3 | 1.7 | 2.0 | 2.4 |
| *d* | 130 | 220 | 490 | 800 | 1400 |
| Log *z* | 0.0414 | 0.1139 | 0.2305 | 0.3010 | 0.3802 |
| Log *d* | 2.11 | 2.34 | 2.69 | 2.90 | 3.15 |

✓✓

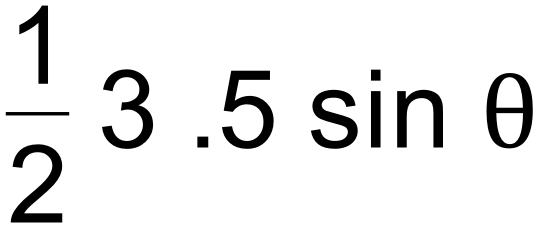
(b)



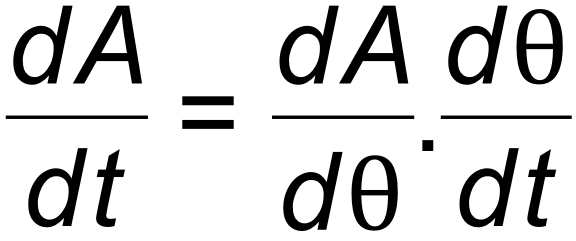
✓✓✓

(c) log *d* = 2 + 3log *z* ✓✓

(d) *d* = 100.z3 ✓✓[9]

15. (a) A =  = 7.5 sin  ✓

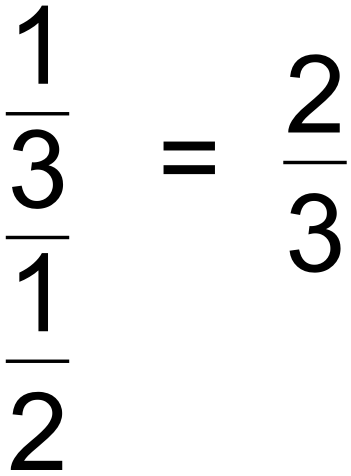
 ✓

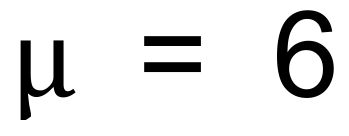
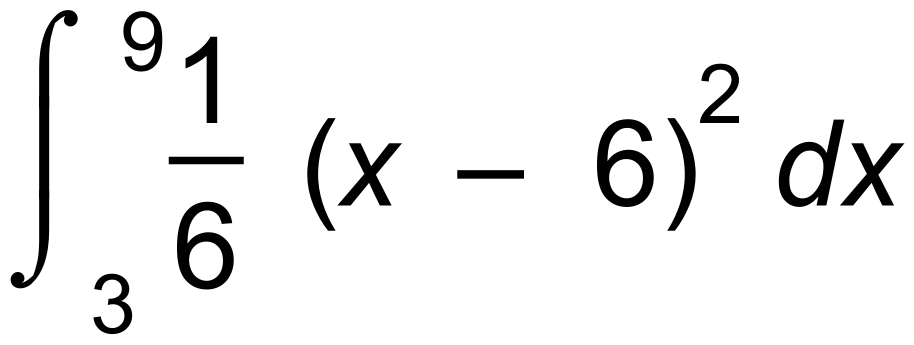
(b) 

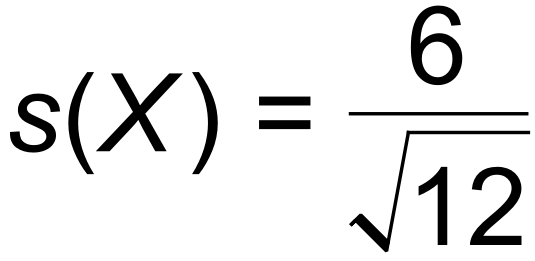
 ✓✓

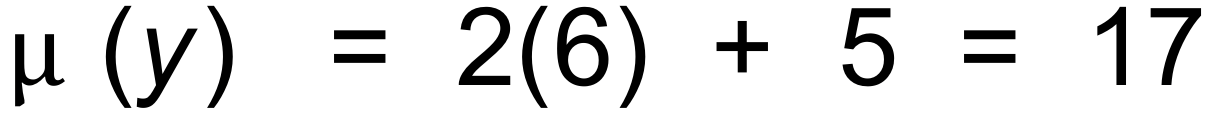
(c) Area has reached a maximum value. ✓ [5]

16. (a)  ✓

(b)  ✓✓

(c)  and Var (X) =  = 3

s(x) =  or  by formula ✓✓

(d)  ✓

s(*y*) = 2 ✓ [7]

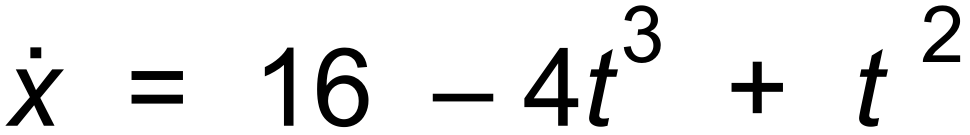
17. (a) P(390 < X < 410) = 0.15852 ✓✓

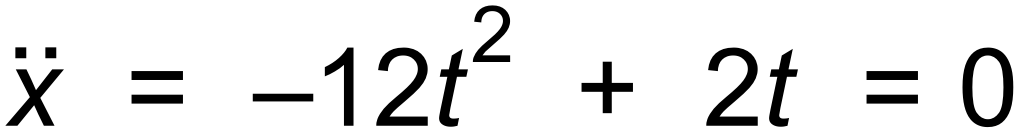
(b) P(X < 400 + *k*) = 0.96 ✓

∴ 400 + *k* = 487.534

∴ *k* = 87.534 ✓

(c) *m* = 810 *s* = 100 ✓✓ [6]

18. (a) 

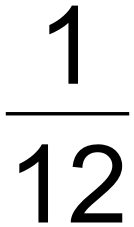
 ✓

∴ *t* = 0 or  ✓

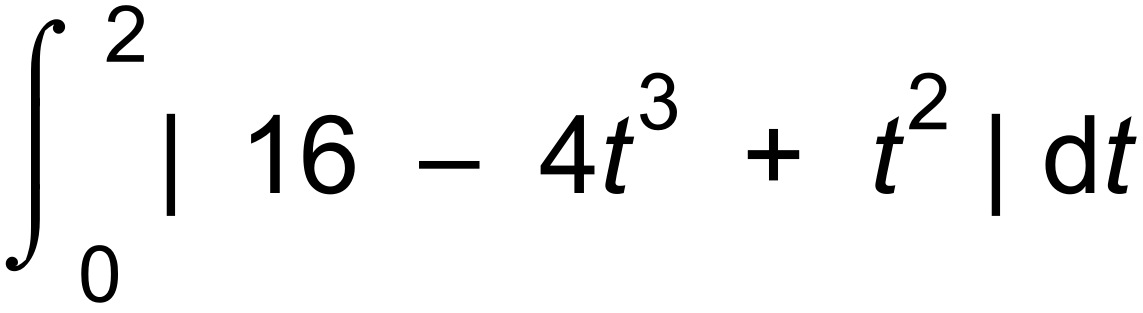
(b) 16 – 4*t*3 + *t*2 = 0 ✓

*t* = 1.68 ✓

(c) –12*t*2 + 2*t* is a maximum when –24*t* + 2 = 0

ie when *t* =  ✓

*x* = 5.33 ✓

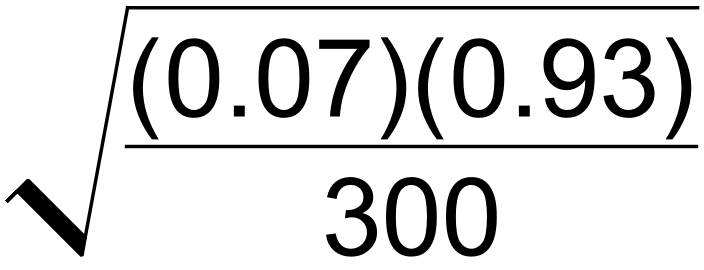
(d)  = 22.323 m ✓✓ [8]

19. (a) Normal curve ✓

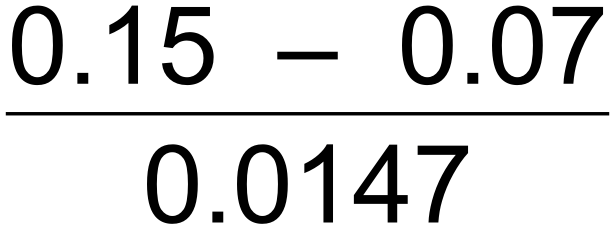
(b) mean = 21 ✓

standard deviation =  = 4.4193 ✓

(c)  = 0.07 ✓

standard deviation =  = 0.0147 ✓✓

(d) This is a proportion of 0.15

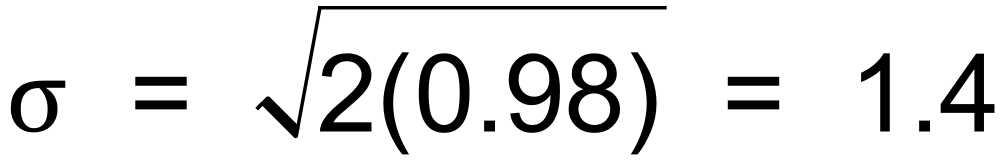
*Z* =  = 5.44

5.44 standard deviations above the mean is very unlikely.

The testing method may need reviewing. ✓✓ [8]

20. (a) Binomial ( 100, 0.02) ✓

 ✓

 ✓

(b)  = 0.0508 ✓✓

(c) *n* = 2000, *p* = 0.02, X = 40

90% interval is 0.0149 to 0.0251 from CAS ✓✓

(d) P(X = 2) = 3C2 (0.9)2(0.1) = 0.243 or from CAS ✓✓

(e) 0.0149 x 2000  30

0.0251 x 2000  50 ✓✓

(f) Interval is from 30 to 50.

Sample 2 is outside. ( 57 > 50 )

Sample 3 is outside. ( 28 < 30 ) ✓✓ [13]